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Experimental Investigation of Panel Divergence at Subsonic Speeds

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An experimental investigation has been conducted of the post-divergence, as well as the onset of divergence, behavior of a flat, rectangular panel at low airspeeds. Comparisons with available nonlinear theory show qualitative agreement for panel strain as well as for the dynamic pressure at the onset of divergence, except for the thinnest panel tested, where experimentally measured strains were substantially higher than those predicted theoretically. It is thought that geometric imperfections were largely responsible for this difference.

Nomenclature

a = panel length
b = panel width
D = $Eh^3/12(1-\nu^2)$, panel flexural stiffness
E = Young's modulus
h = panel thickness
K = dimensionless frequency
M = Mach number
Δp = static pressure difference across the panel
q = dynamic pressure
q_D = dynamic pressure at panel divergence
R_T = $12(1+\nu)\alpha(a/h)^2\Delta T$, non-dimensional temperature differential
ΔT = temperature difference between panel and frame
ΔT_b = temperature difference required to buckle panel
U = air speed
w = panel displacement in z direction
x = chordwise spatial variable
y = spanwise spatial variable
α = coefficient of thermal expansion
ρ = density of air
ρ_m = density of the panel
λ^* = nondimensional dynamic pressure
λ_f^* = nondimensional flutter dynamic pressure
ν = Poisson's ratio
ϵ = strain
ω = cyclic frequency

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I. Introduction

A. The Problem and Its History

FOR sufficiently thin panels static divergence can occur at a lower dynamic pressure than does flutter, usually at subsonic speeds only, and if it does occur, is in most cases a mild type of instability. Emphasis has heretofore been on high Mach number panel flutter. Panel divergence is thought to be understood mathematically, but very little work has been done on experimental panel divergence.[1]

Physically, divergence occurs when the aeroelastic system becomes statically unstable. Theoretically, the linear response of the system grows exponentially with time. In practice, the nonlinear response will reach some static equilibrium diverged condition after some exponential temporal growth. This diverged condition as well as the onset of divergence will be considered here.

One of the most active researchers in the field of panel divergence is Ishii.[2] Mention should be made of the work of Richardson and Johns,[1,3] Dugundji, Dowell, and Perkin,[4] and Sykes and Thomas.[1,3]

B. Discussion of the Important Physical Parameters

Midplane load is one of the most important parameters in divergence. Tension often prevents catastrophic failure in the presence of restrained edges because the restoring force increases nonlinearly faster than deformation. Ishii has attempted to minimize midplane load in his experiments by mounting the trailing edge of the panel without restraints, and applying a constant mid-plane force. He has been quite successful in obtaining quite large-scale deflections (over 1 in.) that way.

Another potentially important parameter is the temperature difference between the panel and the frame supporting the panel. If the panel is warmer than the frame, tending to produce compression, the effect on an unbuckled panel will be destabilizating. For the panel colder than the frame the opposite effect occurs. In most cases, the panel changes temperature more quickly than the panel support. Ventres[5,6] gives some critieria for panel response as a function of temperature in his work. His plot of flutter dynamic pressure vs temperature differential indicates where thermal buckling becomes a problem.

Pressure differential between the exposed surface of the panel and the unexposed surface of the panel is important. Ideally, for the present study, it would be best to have no such differential on the panel, since it interferes with the aerodynamic loading of the panel. Ishii encountered some difficulty in his work in eliminating this pressure differential. He found that it could be made to have a destabilizing effect on the system if the differential was zero at the center of the panel.[2] Also of importance is the presence of a pressure gradient across the surface of the panel. Ishii found this could also have a destabilizing effect on the system. A more complete account of the present work may be found in Ref. 7.

II. Experimental Program

A. Problem Definition

It is desired to investigate the behavior of a clamped aluminum panel with a chord to span ratio, a/b, of 2. To insure that the behavior of the panel would be statically divergent a low Mach number flow regime was selected. Panel thickness was to be varied in an attempt to obtain static divergence. The experiment was conducted in the 3 ft × 5 ft wind tunnel of Princeton Univ.

It is desirable to have the panel as large as possible. The optimum dimensions were 16 in. × 32 in. plus a 1-in. border for clamping the panel on the frame. The panel frame was made of aluminum, the same material as the panels, to minimize the problems of thermal expansion; 1 in. × 2 in. U-sections were used. Here an effort was made to keep the frame shallow so that the thickness of the panel and frame assembly in the tunnel would be minimal, although some thickness was necessary to accommodate the panel instrumentation. The over-all dimensions of the frame were 18 in. × 34 in. × 2 in. A support for a magnetic displacement transducer was incorporated into the panel frame.

B. Boundary Conditions and Panel Natural-Vibration Modes

Provision was made for vibration testing of the model to determine natural modes and frequencies. A magnetic transducer was employed to record dynamic motion of the panel. This, coupled with a graphic level recorder and loudspeaker, was used to find dynamic response of the panel. The dynamic-response bench test was used to gage what boundary conditions had actually been achieved. A small magnetic

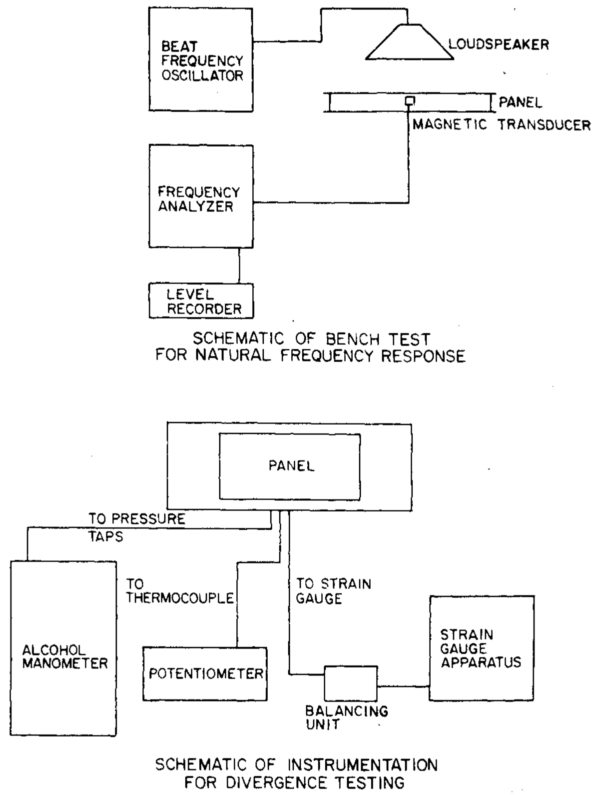

Fig. 1 Schematic of instrumentation.

disk was fastened onto the panel where transducer measurements were to be taken. See Fig. 1.

Several different panels were tested. They were 0.063-in., 0.032-in., 0.020-in., and 0.016-in.-thick aluminum panels. These panels were bonded to the frame with an adhesive.

In constructing the panel-frame model an effort was made to achieve a clamped boundary condition that was called for in the design. The dynamic pressure at which divergence will occur is dependent upon this condition. A bench test of the model was performed to determine what boundary conditions were actually achieved.

Ventres[5,6] gives data about the natural-frequency response of various panel configurations. From his data it is possible to determine theoretically the modes of the panel under consideration.

Nondimensional frequency is given by the expression $K = \omega[\rho_m h b^4/D]^{1/2}$. The data found for the panels considered are given in Table 1.

C. Temperature Effects

The effect of temperature induced strain was considered, as there is an appreciable rise in the tunnel temperature with use of the tunnel. An ironconstantan thermocouple was used to take temperature measurement. The thermocouple was connected near the center of the panel and on the inside of the frame. The lead wires were of iron and were kept to the same

Table 1 Experimental and theoretical panel frequency response data

		0.063 in.		0.032 in.		0.020 in.		0.016 in.	
Mode	K, Ref. 5	cps, theory	cps, exp.	cps, theory	cps, exp.	cps, theory	cps, exp.	cps, theory	cps, exp.
1	24.2	57.4	57	28.8	30	17.9	20	14.4	×
2	30	66.8	66	35.7	36	22.1	26	17.8	×
3	40.7	96.8	95	48.4	50	29.8	32	24.8	25
4	57.2	134	135	68.1	70	40.5	45	34.0	34
5	63	150	150	75.0	77	46.5	50	37.4	38

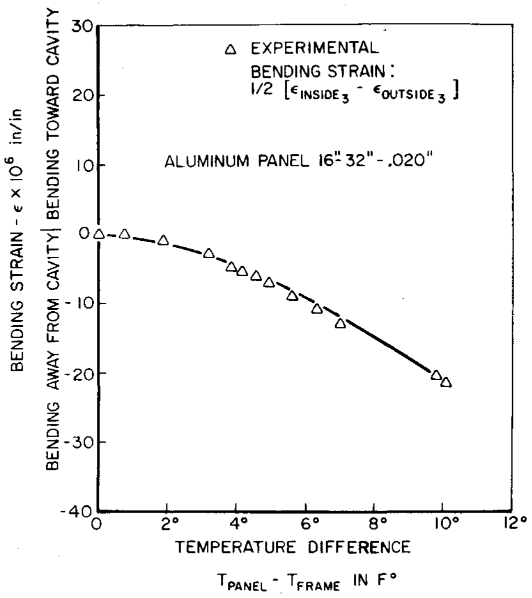

Fig. 2 Strain vs temperature difference.

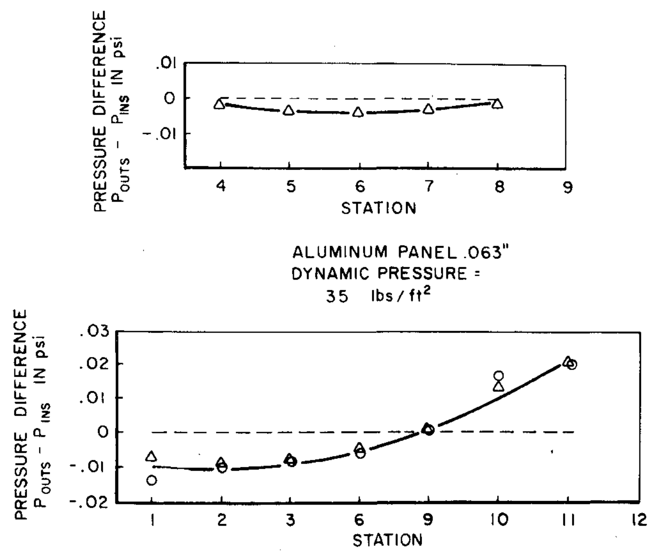

Fig. 4 Pressure difference vs station.

length. A potentiometer was used for the temperature measurement. No reference ice-bath was used since an absolute value of temperature was not important; the significant parameter is the temperature difference between the panel and the frame.

The temperature of the panel was varied by heating the panel with heat lamps. At the same time the temperature difference between the panel and the frame was found from the millivolt potentiometer, and the temperature-induced strain was recorded from the meter of the strain gage apparatus. Typical temperature data obtained can be seen in Fig. 2.

D. Panel Aerodynamic Loading

To achieve good flow from front to back of the panel-frame assembly it was decided to incorporate it into a very thin airfoil with known properties. NACA 16-006 was chosen. The section was split at midchord and the respective parts used as a trailing edge and a leading edge. The over-all thickness of the airfoil was 2.2 in. The airfoil consisted of leading and trailing edges and siderails of wooden materials. All lead wires for instrumentation were carried internally. A smooth finish was applied to the model to decrease roughness. The model spanned the tunnel vertically and was mounted in such a way that it could be rotated slightly. This made it possible to alter the angle of attack of the airfoil without great difficulty. It was found that mounting the model in the center of the tunnel was optimal. See Fig. 3.

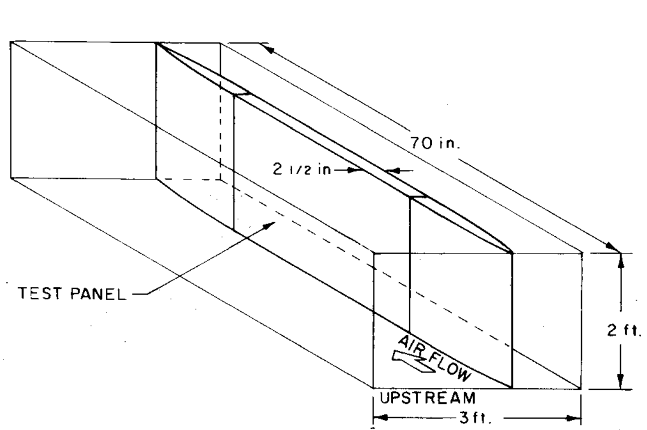

Fig. 3 Model and tunnel configuration.

Mounting structure was kept to a minimum. Those mountings that protruded into the tunnel were aerodynamically faired.

As the testing began it was found that angle of attack of the airfoil was critical and that very slight variation on the angle of attack changed the aerodynamics greatly. A provision was made so that angle of attack could be accurately adjusted.

A special panel was prepared for investigating model aerodynamics. This was a relatively thick panel, 0.063 in. aluminum, with a total of eleven static taps on its surface. Also included were four static taps at various locations inside the cavity of the system.

In testing pressure distributions the dynamic pressure in the tunnel (speed) was varied and the readings of the pressure taps on a manometer were recorded. The angle of attack was varied in the various runs until the most favorable condition was achieved. Several runs were made to establish repeatability of the data.

Typical data that were obtained on pressure distributions are shown in Fig. 4.

E. Divergence Testing

Several panels were tested for divergence. They were 0.063-in., 0.032-in., 0.020-in., and 0.016-in.-thick aluminum

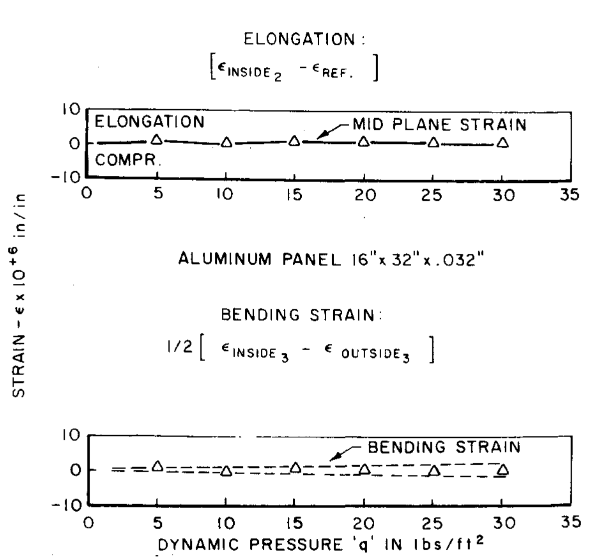

Fig. 5 Strain vs dynamic pressure.

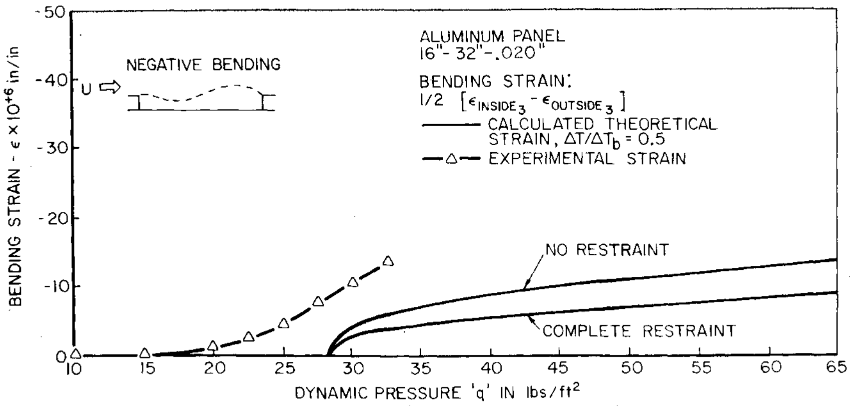

Fig. 6 Bending strain vs dynamic pressure.

panels that had been instrumented with strain gages. Tape was used to prevent crossflow throughout the cavity.

For the divergence testing, strain gage instrumentation was used. The gages at quarter and three-quarter chord were to be used to measure bending of the panel. Essentially, one gage output was subtracted from another to determine this bending strain. This technique discriminates between first-mode bending and second-mode bending. First-mode bending can be detected by adding the outputs of the gages at $\frac{1}{4}$ and $\frac{3}{4}$ chord.

The gage at half chord was employed with a temperature compensating gage to determine the midplane strain of the panel. This technique did not differentiate between bending and elongation, but at $\frac{1}{2}$ chord with second mode bending, the bending strain reading should be zero. Therefore all measured strain will be midplane strain.

Later a gage was located on the outside of the panel at the $\frac{3}{4}$ chord point to measure pure bending in spite of the possibly bad effect that this would have on the aerodynamics. It was found that this had no appreciable effect on the aerodynamics and was a superior technique. Wiring was made as unobstructive as possible.

The strain response of the panels was tested at various dynamic pressures. If midplane strain was not zero for low values of dynamic pressure, angle of attack was readjusted until mid-plane strain reached a minimum for low "q." With that angle of attack fixed, the dynamic pressure was varied over the entire range and the strains recorded for each dynamic pressure. An attempt was made to observe at what values of q divergence occurred.

The potentiometer and the thermocouple were employed concurrently to record the temperature difference between the panel and the frame at the various values of dynamic pressure. Pressure taps were used in conjunction with static taps in the cavity to gage the pressure difference between front and back and inside and outside of the panel.

Strain records vs dynamic pressure for the various panels are shown in Figs. 5–9.

III. Discussion of the Experimental Results

A. Discussion of Boundary Conditions and Panel Natural Modes

The first phase of the experiment involved establishing the boundary conditions of the panel through theoretical-experimental correlation of modal frequencies. These results, presented in Part II, are quite close to the calculated theoretical values for panel response. The lower frequency values are less accurate since the experimental apparatus is less dependable at those values. The results for the 0.020 in. thick panel seem to be slightly high, but they are still quite acceptable.

It may be concluded from these tests that the boundary conditions of the panels were all quite close to the clamped boundary condition that the experimental design called for.

B. Discussion of Temperature Effects

The heat lamp experiment was employed primarily to determine when thermal buckling would occur and if any noticeable effects accompanied it. From Ventres[5,6] one may estimate that thermal buckling will occur for a panel of $a/b = 2.0$ at a value of R_T around 1,000 for typical in-plane support flexibility. Considerably smaller values of R_T can occur if the in-plane support approaches complete restraint. For panels under consideration, see Table 2. It can be seen that the values of R_T for the various panels used permit a temperature difference of 5.5°F before the onset of thermal buckling. In actual experimental runs the temperature difference did not exceed 2.5°F.

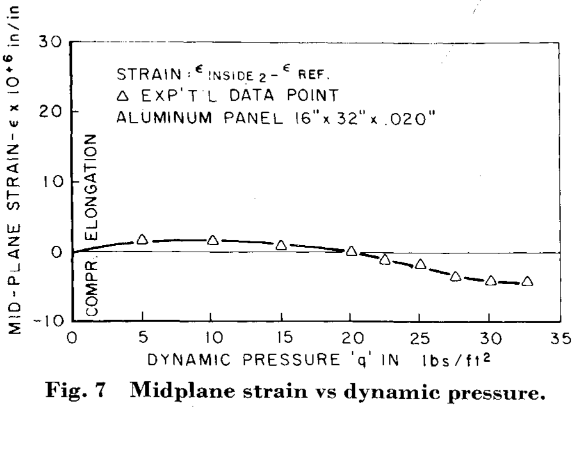

Fig. 7 Midplane strain vs dynamic pressure.

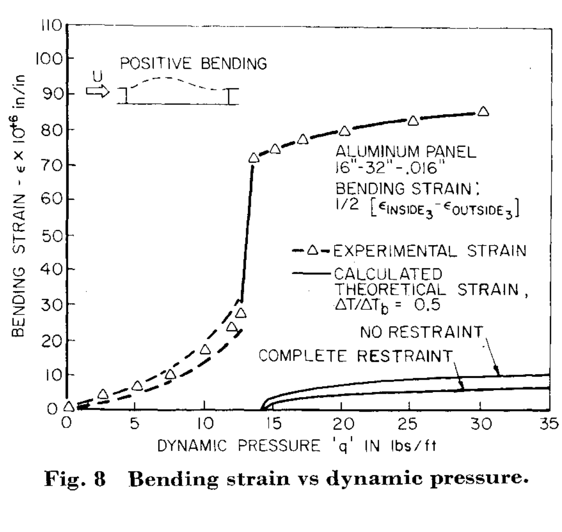

Fig. 8 Bending strain vs dynamic pressure.

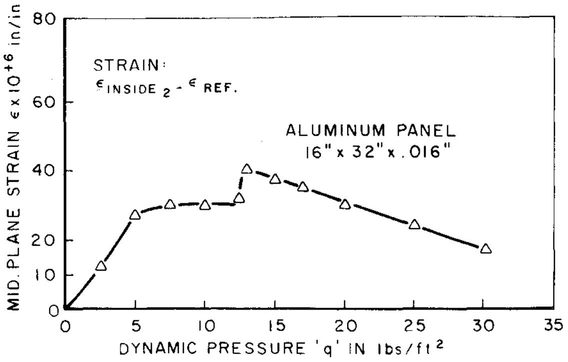

Fig. 9 Midplane strain vs dynamic pressure.

This indicates that thermal buckling per se is not a major problem with these panels. The effect of the temperature difference encountered can be compared to the graphs in Fig. 2. The temperature induced strains in the panel (3 μin.) can be seen to be quite small compared to the values of strains that are encountered in divergence. (See Figs. 5–9.) However the temperature differential may still have decreased the effective stiffness of the panels. Unfortunately, no experimental assessment of in-plane support flexibility per se was made. That is, the panels were not tested to the point of buckling. In retrospect such a test would have been valuable in order to provide a basis for obtaining a theoretical estimate of thermal stress effects on the divergence behavior. See later discussion in Sec. D.

C. Discussion of the Model Aerodynamics

The initial wind-tunnel testing involved establishing the pressure loading that existed on the panel. Figure 4 shows a chordwise distribution of pressure below, and a spanwise distribution above. It is desirable to have the non-dimensional static pressure loading, P, less than 2000 where $P = \Delta p b^4/Dh$. An effort was made to minimize this variation.[5]

The data shown in Table 3 were found for the panels under consideration.

D. Discussion of the Divergence Tests

A panel of 0.063 in. thickness was first tested for divergence. Angle of attack was not as finely adjusted as it might have been. Also, strain gages on the inside of the panel, locations 1 and 3, were used. No gages on the outside of the panel were used.

Using linearized potential theory and a Galerkin's solution as in Refs. 5 and 6, one finds that the onset of divergence is predicted at $q_D = \lambda_D{}^*D/2a^3$ where $\lambda_D{}^* = 3470$ for $a/b = 2.0$ and $\lambda^* \equiv \rho U^2 a^3/D$. Thus we may find the divergence dynamic pressure for the panels under consideration; see Table 4.

Furthermore, one may calculate displacement ratio, w/h where w is the panel displacement and h is the panel thickness, vs λ^*. See Fig. 10. By assuming a second mode divergence, which the experimental investigation confirms, theoretical bending strain may be found as a function of dynamic pressure for the panels under consideration from the well-known formula, $\epsilon_x = \pm(h/2)\partial^2 w/\partial x^2$.

The relevant formulas employed were

$$w/h = a_2[1 - \cos(2\pi y/b)][\cos(\pi x/a) - \cos(\pi y/b)]$$

where a_2 is determined from

$$[C_{22} + \lambda^* Q_{22}]a_2 + B_{2222}a_2{}^3 = 0$$

and, for $a/b = 2$, $C_{22} = 2.63 \times 10^4$, $Q_{22} = -7.58$, $B_{2222} = 1.567 \times 10^5$ (in-plane edges fully restrained), or $= 0.58 \times 10^5$ for zero in-plane edge restraint. The divergence dynamic pressure is given by

$$\lambda_D{}^* = C_{22}/-Q_{22}$$

Referring to theory for the onset of divergence for a panel 0.063 in. thick, one would find the value for divergence dynamic pressure to be quite large, 1700 psf. Of course, the wind tunnel is not capable of such performance. The response of the panel should have been smaller than it was. It would have been if the angle of attack had been adjusted more carefully and a strain gage had been used on the outside of the panel. Some inaccuracy might have been because of thermal effects on the strain gage.

After the 0.063 in. thick panel had been tested, a 0.032-in. thick panel was tested. Here a strain gage was used on the outside at station 3 as well as on the inside. This gave a pure bending reading of greater accuracy. It was apparent from comparison of the results from gages 1 and 3 with and without a gage located outside on the panel, that the panel strain response was not adversely affected by the location of a strain gage in the airstream.

Again referring to theory, one may find that a panel of thickness 0.032 in. will diverge at a "q" of 224 psf. This is out of the tunnel's reach. The data shows there is some dynamic strain in the bending of the panel (response to "noise") but it is quite small.

The peak-to-peak dynamic response of the panel was of the order of 4 μin. per in. Midplane strain seemed to remain constant at zero. The accuracy of the midplane reading was less than that of the bending readings because only one active gage was used for that measurement. Dynamic strain would be more difficult to determine with that arrangement. No static bending strain was found in this panel. None had been expected. See Fig. 5.

A panel of thickness 0.020 in. was tested next. The technique was essentially the same as it had been before. Gages were placed inside the panel as before and measurements were taken using only them. Then a gage was applied outside the panel at location 3 and a check made to see if strain response was altered with its installation. No alteration was found.

Because of the thinness of this panel and the correspondingly low bending stiffness, D, some problems with sag were anticipated with this panel. None were encountered in practice. This might have been because the panel-frame arrangement was mounted vertically in the wind tunnel and sag had less opportunity to develop, than with other configurations.

The theoretical prediction of the value of dynamic pressure for which divergence would occur for a panel of thickness 0.020 in. was 56 lb/ft^2. The data in Fig. 6 shows that a marked increase occurs in the bending strain of the panel in the vicinity of 25 lb/ft^2.

The behavior of midplane strain is similar to what might be expected. The values are quite small, but show an increase with the onset of divergence. Because only one active gage was used, the measurement is not precisely of midplane elongation but rather of midplane elongation on one side of the panel. If divergence is in the second mode, as the experiment indicated, then theory predicts that bending strain at midchord will be zero. Then it may be inferred that any strain recorded at midchord will be mid-plane tensile (or compres-

Table 2 Panel temperature response

Panel thickness	$R_T/\Delta T$	Max ΔT in testing °F	Max R_T
0.063	12.4	1.8	22
0.032	24.8	1.8	45
0.020	123	2.5	315
0.016	192	2.1	400

sive) strain due to midplane stresses. Installation of another gage on the outside of the panel for more precise measurements was not feasible because of the necessity for long lead wires that would have interferred with the flow over the panel much more than one gage used for bending. In addition, the primary interest was the bending of the panel.

Because of the success of the experiment using a panel of 0.020-in. thickness, it was decided to attempt to use a panel of 0.016-in. thickness. As already shown, the bending stiffness, which changes as the cube of thickness, is one-half of that of the 0.020-in. panel. Here again, sag was anticipated to be a problem as was handling such a thin panel. The problem of thermal buckling did not seem to worsen significantly and was rather under control. The technique of testing was similar to that used for the panel 0.020-in. thick. Care was taken to keep the panel as flat as possible; however, measurements indicated that the panel deviated from flatness by as much as three panel thicknesses, 0.048 in., at stations 1, 2, and 3. This indicates that the panel might be expected to behave more like a curved panel than a flat panel. See Ref. 9 for a discussion of the flutter behavior of curved panels. The predicted theoretical value for divergence dynamic pressure was 29 lb/ft².

Figure 8 shows the experimental data for this panel. The panel has a dynamic strain response which builds up to ± 7 μin. per in. just before the onset of divergence. Divergence is quite dramatic. It occurs at a "q" of 12.5. The direction of the bending will change with very small changes in angle of attack. There is a slight falling off of dynamic response after the onset of divergence.

Sag was found to be a problem with this panel. The panel was not absolutely flat after it had been installed, whereas all the other panels that were tested were essentially flat. This might have accounted in part for the very large strain readings that were obtained. The initial curvature of the 0.016-in. thick panel might explain the large values of strain. This is typical of the flutter response of curved panels,[9] and similarly might be important for static divergence. The sag did not seem to affect the behavior of the panel in any other way. If a panel of thickness 0.018 in. had been available, it would have been the optimum thickness for this experiment. A thinner panel begins to experience sag and this is undesirable.

Midplane strain was fairly large for the 0.016 in. panel as shown in Fig. 9. Qualitatively its behavior is similar to that of the 0.020 in. panel, though it is less well behaved.

Because the theoretical and experimental results for q_D differ substantially quantitatively, it was decided to reduce the linear stiffness of the panel in the theoretical calculation in order to make comparisons with experiment in the post-divergence condition. This would correspond to a temperature difference of one-half the buckling value which is not unreasonable in view of the measured temperature difference of 2°F and the estimated buckling temperature difference of 5°F.

This somewhat arbitrary reduction allows a meaningful comparison between theory and experiment for the post-divergence regime. See Figs. 6 and 8. With this modification the agreement is qualitatively good for the 0.020-in. panel but very poor for the 0.016-in. panel, the latter probably due to the curvature of the panel as a result of substantial geometrical imperfections.

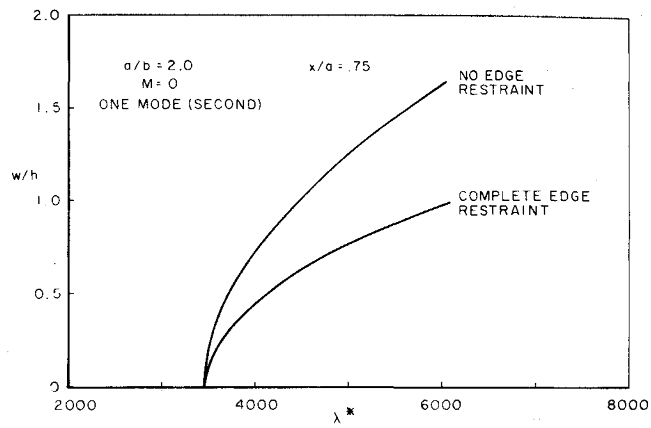

Fig. 10 Panel deflection ratio vs nondimensional dynamic pressure.

IV. Critique and Implications of the Experiment

Predicting where divergence will occur does not seem to be difficult, and reasonably accurate results can be expected. Theoretical calculation of strain distribution after divergence correlates qualitatively with measured strain after divergence if the panel is essentially flat. In this experiment, sag that gave the 0.016 in. panel a curvature of some three panel thicknesses significantly altered the experimental strain results from those expected for a perfectly flat plate.

Boundary conditions are important and in some cases evaluating them may require good judgment. Aspect ratio of the panel under consideration will be important in determining in which mode divergence will occur and what the characteristic length of the static wave will be. It seems that the number of the mode in which a panel will diverge will be in relation to the chord to the span ratio of the panel. In this experimental investigation, panels with an $a/b = 2.0$ were found to diverge in the second mode, or in a shape similar to a full sine wave.

The shape of the curves of strain vs dynamic pressure was slightly different from what was expected on the basis of theory. A sharper rise in strain had been anticipated at divergence. In view of the results obtained by Thomas this is not alarming. Some of the distortion in the curve at a low dynamic pressure is probably due to a pressure difference across the panel. Divergence was observed and was found to behave substantially as predicted.

Thermal stresses may also have been a problem though it was possible to keep the panels from thermal buckling prior to divergence. Thicker panels could be tested at higher Mach number which would be less sensitive to temperature and pressure differentials.

By itself, the effect of divergence on structural stiffness is yet to be fully assessed and this is where divergence is important. As a panel diverges, the nonlinear increase in midplane stress finally brings static equilibrium in a diverged state. This build-up of stress may have an important effect on the structure.

If structural properties are changed with divergence, then the analysis for higher speed instabilities for response to flow

Table 3 Nondimensional panel static pressure loading, P

Thickness, in.	$P/\Delta p$	Max Δp encountered in divergence testing	Max P
0.063	4400	0.005 psi	22
0.032	68300	0.005 psi	340
0.020	448000	0.005 psi	2240
0.016	1100000	0.005 psi	5500

Table 4 Theoretical dynamic pressure for onset of divergence

Panel thickness, in.	q_D lb/ft²
0.063	1700
0.032	224
0.020	56
0.016	28

"noise" might have to take this change into account. If strength is changed with divergence and panels are designed to carry loads, a change in strength caused by divergence would imply the necessity for a design that would take this into account.

V. Conclusions and Recommendations

On the basis of the experimental investigations of panel divergence conducted, the following may be concluded:

1) An experimental technique has been developed to determine divergence behavior, strain response and the dynamic pressure at which divergence begins, for clamped panels.

2) Divergence occurred roughly at the dynamic pressure that linearized potential theory and a one-mode Galerkin's solution indicates.

3) Postdivergence strain response for the 0.020-in. panel tested is qualitatively as predicted by the linearized potential theory and a one-mode Galerkin's solution, including nonlinear structural effects.

4) Sag and the corresponding curvature of panels greatly increases the post-divergence strain response of thin panels.

5) The mode in which divergence occurs is in relation to the length/width of the panel, namely a panel with length/width = 2.0 diverges in a shape similar to a full sine wave in the chordwise direction.

In view of the experimental investigation that has been conducted, the following recommendations are made:

1) An analysis should be made of the strain response of clamped, curved panels in the post-divergence region and an experimental investigation should be made of clamped, curved panels in the post-divergence region.

2) Divergence mode shape should be thoroughly investigated for panels of various chord to span ratios.

3) Postdivergence as well as predivergence flow noise response of clamped panels should be investigated experimentally.

4) A similar experiment at low supersonic speed for panel flutter would be extremely valuable and would alleviate the necessity of using such thin panels as those employed here which are rather sensitive to thermal stresses and pressure differentials.

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